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TURBULENT TEMPERATURE FLUCTUATIONS IN LIQUID METALS

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Abstract—Examination of experimental data for the spectral distribution of velocity (u and v) and temperature (θ) fluctuations in the fully turbulent region of heated pipe-flow has suggested a schematic representation which incorporates the essential features. Evidence is cited to suggest that the "$-v\theta$" correlation coefficient maintains higher values than the "uv" coefficient at wave-numbers in the inertial subrange. The theory of Batchelor, Howells and Townsend, and limited evidence from experiments in mercury, then suggests the form of the "θ^2" spectra and "$-v\theta$" cross-spectra in liquid metals. From this information, a limiting Peclet number is deduced, above which the correlation coefficient of v and θ should be a fairly weak function of Pe alone.

An attempt to check this inference from published data for the RMS level of temperature fluctuations, and for the turbulent Prandtl number, proves inconclusive, because many of the correlation coefficients so estimated have values greater than unity. It is concluded that all these results for $\tilde{\theta}$ must therefore be in error. However, since there is no evidence of very low correlation coefficients, they almost certainly lie in the range $0.5 \gtrless 2$ over a large proportion of the radius. Thus $\tilde{\theta}$ can be estimated for any fluid in which the fluctuations are induced by uniform heating, at least to within a factor of 2, using the analysis of this paper.

NOMENCLATURE

a,	pipe radius;
Cp,	specific heat of fluid;
$E_x(ak_1)$,	power spectral density of $X(=u^2, uv, \text{etc.})$;
f,	friction factor, $\equiv 2\left(\dfrac{u_\tau}{U_b}\right)^2$;
k,	three-dimensional wave-number;
k_1,	one-dimensional wave-number;
L,	length-scale of energy containing eddies;
Nu,	Nusselt number, $\equiv \dfrac{2\dot{q}''_w a}{(T_w - T_b)\rho Cp\alpha}$;
Pr,	Prandtl number, $\equiv \dfrac{\nu}{\alpha}$;
Pr_t,	turbulent Prandtl number, $\equiv \dfrac{\varepsilon_m}{\varepsilon_h}$;
Pe,	Peclet number, $\equiv Re\,Pr$;
$\dot{q}''$,	radial heat flux at r;
$\dot{q}''_w$,	wall heat-flux;
$R_x(ak_1)$,	cross-spectral correlation coefficient of $X(=uv \text{ or } v\theta)$, $R_{uv} \equiv \dfrac{E_{uv}}{(E_{u^2}E_{v^2})^{1/2}}$;
$R_{uv}, R_{v\theta}$,	overall correlation coefficient, $\equiv \dfrac{\overline{uv}}{\tilde{u}\tilde{v}} \Big/ \dfrac{\overline{v\theta}}{\tilde{v}\tilde{\theta}}$;
R_∞,	value of correlation coefficient $R_{v\theta}$ as $Pe \to \infty$;
r,	radial position;
St,	Stanton number, $\equiv \dfrac{\dot{q}''_w}{\rho Cp U_b(T_w - T_b)}$;
T_w,	wall temperature;
T_b,	bulk fluid temperature;
t_τ,	"friction" temperature, $\equiv \dfrac{\dot{q}''_w}{\rho Cp u_\tau}$;
U,	time-averaged axial velocity;
U_b,	bulk velocity;
U_m,	maximum velocity (at $r = 0$);
u_τ,	friction velocity;
u,	fluctuating component of axial velocity;
v,	fluctuating component of radial velocity;
w,	fluctuating component of circumferential velocity;
y,	distance from wall, $\equiv a - r$.

Greek symbols

α,	thermal diffusivity of fluid;
β,	proportion of θ^2 power spectrum at $k_1 > \eta_c^{-1}$;
γ,	proportion of v^2 power spectrum at $k_1 > \eta_c^{-1}$;
ε,	rate of dissipation of turbulence energy;
ε_h,	eddy diffusivity of heat;
ε_m,	eddy diffusivity of momentum;
θ,	fluctuating component of temperature;
θ_∞,	fluctuating component of temperature in high Prandtl number fluid at the same Re (i.e. as $Pe \to \infty$);
η,	Kolmogoroff length scale, $\equiv \left(\dfrac{\nu^3}{\varepsilon}\right)^{1/4}$;
η_c,	Kolmogoroff length scale of temperature fluctuations, $\equiv \left(\dfrac{\alpha^3}{\varepsilon}\right)^{1/4}$;
ν,	kinematic viscosity of fluid;
ρ,	density of fluid.

Superscripts

+, { distance normalized by $\frac{\nu}{u_\tau}$; velocity normalized by u_τ;

$\bar{}$, time-averaged.

INTRODUCTION

UNDER most laboratory conditions, turbulent heat transfer in liquid metals is dominated by molecular conduction and an accurate description of the turbulent processes is not essential in predicting temperature distributions. With the advent of sodium-cooled fast reactors however, the combination of low Prandtl number with sufficiently high Reynolds numbers to ensure the dominance of turbulent diffusivity becomes a situation of some importance. The prediction of the attenuation of thermal transients in large sodium volumes and the interpretation of instrumentation for detecting fuel element blockages provide just two reasons for needing to understand the mechanism of turbulent heat transfer under these conditions.

The traditional method of calculation arrives at the turbulent diffusivity of heat by way of its ratio to the turbulent diffusivity of momentum, through the so-called turbulent Prandtl number, $Pr_t \equiv \varepsilon_m/\varepsilon_h$. Several sets of experimental data for Pr_t in the flow of liquid metals in a heated pipe are now available but, partly because these were acquired at comparatively low Reynolds numbers, where molecular and turbulent diffusivities are comparable, there are considerable discrepancies between results for the same values of Re and Pr. It is shown here that large variations in Pr_t usually lead to only small ones in the parameter, $-\overline{v\theta}/u_\tau t_\tau$, which expresses the ratio of the turbulent radial heat-flux to the heat-flux at the wall. Conversely, small differences in the turbulent conditions between experiments must be expected to give rise to discrepancies in the results for Pr_t, even in the absence of measurement errors.

Since the radial variation of Pr_t in pipe-flow is so poorly defined, the likelihood of success in applying values derived from simple flows to the complex, possibly recirculating flows of interest in fast reactors is in any case remote. The only mitigating factor is that both theory and experiment suggest that $Pr_t \to Pr_{t\infty}$, a value common to all Pr, as $Re \to \infty$. Further research should therefore define a "local" Reynolds number above which $Pr_t = Pr_{t\infty}$ is a valid approximation, but this Re may well be too high for many cases of interest.

Rather than try to find some greater generality in the turbulent Prandtl number concept, it may therefore be more fruitful to compare, not the diffusivities of heat and momentum, but the R.M.S. values of the turbulent temperature and axial velocity fluctuations, and their correlations with the radial velocity fluctuation, for it is these correlations that give rise to the radial heat and momentum fluxes. It is shown in the next section that consideration of what is known of the spectral distribution of the turbulent fluctuations suggests that the correlation coefficient, $-\overline{v\theta}/\tilde{v}\tilde{\theta}$, will not vary greatly with Re or Pr. To test this inference in the following sections, all the available data for the temperature fluctuations in various media in pipe-flow will be examined. It will be shown in Section 3 how Pr_t is related to $-\overline{v\theta}/u_\tau t_\tau$ and then, in Sections 4 and 5, how the $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ correlation coefficient may be derived.

2. DEPENDENCE OF THE HEAT-FLUX CORRELATION COEFFICIENT ON REYNOLDS AND PRANDTL NUMBERS

Direct measurements of the correlation coefficient, $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ in pipe flow, are limited to air. Such measurements have been made by Ibragimov *et al.* [1], Bourke and Pulling [2], Bremhorst and Bullock [3] and Lawn and White [4]. However, the last two references also present data for the one-dimensional spectral distribution of $v\theta$, v^2 and θ^2 and these are supplemented by the measurements of Fulachier and Dumas [5] in a boundary layer. Data for u^2, v^2 and uv are found in papers by Lawn [6] and Bremhorst and Walker [7]. It is to this information in conjunction with the theory of Batchelor *et al.* [8] that we turn to anticipate the behaviour in low Prandtl-number fluids.

The picture which emerges for $Pr \sim 1$ from the above data is that the θ^2 and u^2 distributions (or more particularly, the θ^2 and $u^2+v^2+w^2$ distributions [5]) are very similar throughout the spectrum at all radial positions. In the turbulent core region ($y^+ > 100$ say), for which Fig. 1(a) is a schematic representation of the spectra, the maximum contribution to the variance of these fluctuations occurs at a wave-number $k_1 \sim 2\pi a^{-1}$, while that for v^2 occurs at $\sim 6\pi a^{-1}$. [This can be seen from plots of $(ak_1)E(ak_1)$ vs $\log(ak_1)$, the area underneath which, in a given wave-number range, is proportional to the energy contained.] Above this wave-number, the correlation between u and v disappears rapidly because the turbulence is approaching isotropy, even though $E_{uv}(ak_1) = 0$ does not seem to be a necessary condition for some degree of local isotropy in wave-number space (Bradshaw [9]). At still higher wave-numbers, $k_1 > 0.1\eta^{-1}$, where η is the Kolmogoroff length scale $(\nu^3/\varepsilon)^{1/4}$, on the evidence of Boston and Burling [10] and Lawn [6], and sooner if $Re < 3\times10^4$, the viscous dissipation region is entered and there is a sharp spectral cut-off in u^2, v^2 and θ^2. By contrast with uv, however, the $v\theta$ correlation falls off less rapidly for $k_1 > 6\pi a^{-1}$. This behaviour is explained by the fact that local isotropy of the turbulent motion places no constraint upon $E_{v\theta}(ak_1)$. Indeed, Wiskind [11] has measured $-\overline{v\theta}/\tilde{v}\tilde{\theta} = 0.48$ in approximately isotropic, grid-generated turbulence under a uniform temperature gradient. This value is almost identical to the overall coefficients measured away from the centre-line in pipe-flow [2–4] and shows that fairly uniform isotropic eddies make a similar contribution to heat transport to those with a variety of scales in a strain field. This conclusion is reinforced by the comparative lack of variation in $R_{v\theta}(ak_1)$ $(0.6 > R_{v\theta} > 0.3)$ in the range $0.2 < ak_1 < 20$ in a pipe at $r/a = 0.5$ for example [3].

Turning now to $Pr \equiv \nu/\alpha \ll 1$, the theory of Batchelor *et al.* [8] indicates that the inertial wave-number range

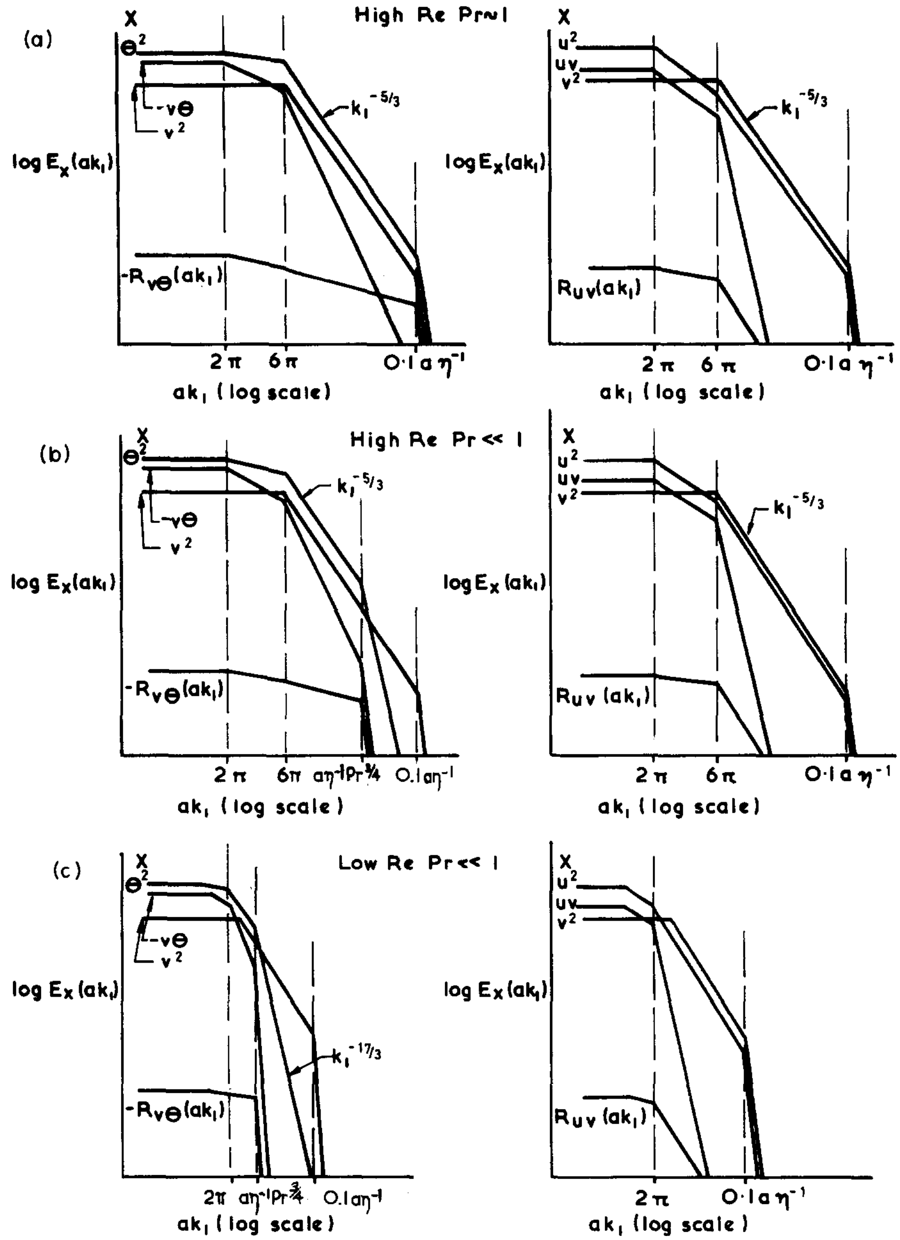

FIG. 1. Schematic representation of the behaviour of one-dimensional power spectra; $\eta = (\nu^3/\varepsilon)^{1/4}$.

$L^{-1} \ll k \ll \eta^{-1}$ (where L is the length-scale of the energy-containing eddies), which has been interpreted above as $2\pi a^{-1} < k_1 < 0.1\eta^{-1}$, should be split into two to describe the temperature fluctuations, as shown in Fig. 1(b). In the "inertial-convective" subrange, $L^{-1} \ll k \ll \eta_c^{-1}$ where $\eta_c = (\varepsilon/\alpha^3)^{-1/4} = \eta Pr^{-3/4}$, the temperature and axial velocity fluctuations continue to be similarly distributed and follow a $k^{-5/3}$ law, while in the "inertial-diffusive" subrange, $\eta_c^{-1} \ll k \ll \eta^{-1}$ the spectrum of temperature is directly affected by the thermal diffusivity of the fluid and so there is a more rapid cut-off due to "conductive-smearing". The only evidence for the validity of this concept is to be found in the theses of Hochreiter [12] and Flaherty [13], which present the results of a recent series of experiments in mercury at Purdue University. Flaherty's data for $Re = 5 \times 10^4$ at $r/a = 0.6$ show cut-off in the velocity spectrum at $ak_1 \sim 50$ and in the temperature spectrum at $ak_1 \sim 20$. These results are confirmed by Hochreiter's data for $r/a = 0.3$ and 0.7. Calculations for this position, Reynolds number and Prandtl number (0.02), based on the data of [6], yield $a\eta^{-1} = 1.6a^{+3/4} \sim 350$ and $a\eta_c^{-1} \sim 20$. Thus it appears that conductive smearing does set in at $k_1 \sim \eta_c^{-1}$ and that once more the velocity spectrum cuts-off at $k_1 \sim 0.1\eta^{-1}$. The reason for the order of magnitude difference in the form of these limits is not clear. That this difference exists is supported by the fact that some workers [5, 14] have found a tendency for the normalized temperature spectra to be higher than the normalized velocity spectra at wave-numbers $k_1 > 0.1\eta^{-1}$, when $Pr \sim 1$ and thus $\eta_c^{-1} > 0.1\eta^{-1}$.

Fuchs [15] has in fact measured temperature spectra in sodium at $Re = 1.4 \times 10^5$. Calculations suggest cut-off at $ak_1 \sim 20$ again (the Peclet number being the same as in the Purdue work) but as the measurements extend only to $ak_1 \sim 0.5$, no information on the conduction cut-off may be gleaned from them.

The above discussion suggests that in liquid metals the spectra of v^2, θ^2 and $v\theta$ will be similar to those in air up to $k_1 \sim \eta_c^{-1}$. Above this wave-number, only v^2 of these three will continue to make an appreciable contribution to its total variance or covariance, and provided $a\eta_c^{-1} > 6\pi$, even this will be small. Translating this condition using [6] again, we expect that $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ will be a very weak function of $(a^+Pr)^{3/4} \propto Re^{0.65} Pr^{0.75}$, for $a\eta_c^{-1} \simeq 1.6(a^+Pr)^{3/4} > 6\pi$. To a good approximation therefore, it should be a very weak function of Peclet number alone for $Pe > 10^3$, provided $Pr \leqslant 1$. This condition translates to $Re > 2 \times 10^5$ for sodium and $Re > 5 \times 10^4$ for mercury. As this Peclet number limit is approached from above, contributions to the θ^2 spectrum which were previously significant, are smeared out, while those to the v^2 spectrum are not. Quantitative consideration of this process in the Appendix provides upper and lower bounds on the variation of the overall correlation coefficient of +27 and −38% for $Pe > 2 \times 10^3$. To describe the situation when $Pe < 10^3$, greater knowledge of the spectral functions is required and since at low $Re(< 5 \times 10^4$ say), the low wave-number end of the hydrodynamic spectra is also modified, somewhat as in Fig. 1(c) on the evidence of [4], predictions for this regime are impossibly difficult. Nevertheless, large variations in the overall correlation coefficient, $-\overline{v\theta}/\tilde{v}\tilde{\theta}$, would not be expected at any Re for $Pr < 1$, and this is the inference to be tested in the succeeding sections.

3. VARIATIONS OF THE NORMALIZED RADIAL HEAT-FLUX WITH REYNOLDS AND PRANDTL NUMBERS

No direct measurements of $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ have been made in a liquid metal, so it is necessary to infer it from the turbulent component of the normalized radial heat-flux, $-\overline{v\theta}/u_\tau t_\tau$, and the data for $\tilde{v}/u_\tau$ and $\tilde{\theta}/t_\tau$, which will be examined in the next section.

The total normalized radial heat-flux under fully-developed conditions with negligible property variations and uniform wall heat-flux may be determined from the energy equation to be:

$$\frac{\dot{q}''}{\dot{q}''_w} = -\frac{\alpha}{u_\tau t_\tau}\frac{\partial T}{\partial r} - \frac{\rho C p \overline{v\theta}}{\rho C p u_\tau t_\tau} = \frac{2}{ar}\int_0^r \frac{U}{U_b} r\,\mathrm{d}r. \quad (1)$$

Using empirical expressions to describe previous measurements in pipe flow [16] of the velocity defect law and maximum-to-bulk velocity ratio, and the Blasius correlation for friction factor to give U_b/u_τ, this may be translated thus:

$$\frac{\dot{q}''}{\dot{q}''_w} = \frac{2}{ar}\int_0^r \left\{\frac{U_m}{U_b} - \frac{u_\tau}{U_b}\left(\frac{U_m - U}{u_\tau}\right)\right\} r\,\mathrm{d}r$$

$$\simeq \frac{2}{ar}\int_0^r \left\{1 + \frac{1.1}{\log_{10} Re} - \frac{u_\tau}{U_b}\left(8.0\frac{r^2}{a^2}\right)\right\} r\,\mathrm{d}r$$

$$\simeq \frac{r}{a}\left\{1 + \frac{1.1}{\log_{10} Re} - \frac{0.8}{Re^{0.125}}\left(\frac{r}{a}\right)^2\right\}, \quad (2)$$

where U_m is the maximum velocity. This equation expresses the "bowing" of the heat-flux profile in terms of the departure of the velocity profile from uniform, as the Reynolds number is reduced from infinity. At $Re = 10^4$, which is the lower limit of validity of the empirical correlations, $\dot{q}''/\dot{q}''_w = 0.606$ at $r/a = 0.5$. Near the wall, the expression is invalid because the velocity defect law does not apply there.

The proportion of the total heat-flux which is due to turbulent transport may be evaluated if values for the eddy-diffusivity of heat are available. Approximately 80% of the flow in the core-region, according to Koo and Wade [17] and many others, is reasonably well described by an eddy-diffusivity of momentum distribution:

$$\frac{\varepsilon_m}{\nu} \simeq 0.07\frac{au_\tau}{\nu}, \quad (3)$$

which using Blasius again becomes:

$$\frac{\varepsilon_m}{\nu} \simeq 0.007 Re^{0.875}. \quad (4)$$

Provided some similar correlation has been used by the experimenters in interpreting their results, only data for Pr_t are further required to allow the turbulent proportion to be evaluated from:

$$\frac{-\rho C p \overline{v\theta}}{\dot{q}''} = \frac{\varepsilon_h}{\alpha + \varepsilon_h} \simeq \frac{0.007 Re^{0.875}}{(Pr_t/Pr) + 0.007 Re^{0.875}}. \quad (5)$$

The parameter $-\overline{v\theta}/u_\tau t_\tau$ for $r/a < 0.8$ and $Re > 10^4$ is thus given as the product of equations (2) and (5). For $Re > 2 \times 10^5$, the Blasius correlation is a poor approximation but the errors incurred are not significant in this context.

Several sets of data for Pr_t in liquid metals have recently been published [12, 15, 18–20] and these are plotted in Fig. 2, together with some representative results for air [21, 22] and a high Prandtl number fluid [22]. All of these are for the arbitrary radial position, $r/a = 0.5$, except for the measurements of Sheriff *et al.* [20], which were deduced from diffusion from a source on the centre-line. Also plotted are the correlations of Isakoff and Drew [23] and Subbotin *et al.* [24], which are based on measurements in mercury at this radial position, and of Dwyer [25], which was deduced. We note that the sodium results show a fairly consistent trend and moreover that Pr_t appears to tend to the high Pr results as $Re \to \infty$. In fact $Pr_t \sim 1$ is a reasonable approximation for $Re > 2 \times 10^5$, which is consistent with the predictions of the last section regarding the invariance of the turbulence structure affecting transport. The mercury data are less consistent but, somewhat surprisingly, appear to vary in the same way with Re as the sodium data. Distortion of the velocity and temperature fields by buoyancy forces [26, 27] in some of these experiments may be responsible for the inconsistency. Only Dwyer's correlation has the asymptotic behaviour at high Reynolds number expected from the present theory. This correlation is based on the fit of an eddy diffusivity model to overall heat-transfer data which should be more reliable than the temperature profiles from which most of the point data were derived.

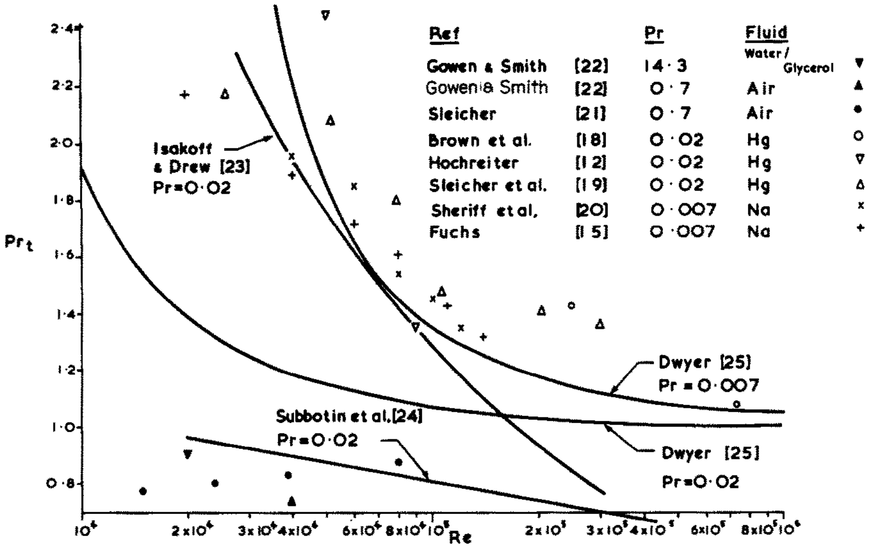

FIG. 2. Turbulent Prandtl number variation with Reynolds number for various fluids. ($r/a = 0.5$, where radial variation given.)

Putting the Pr_t data into equation (5) and also using equation (2) yields $-\overline{v\theta}/u_\tau t_\tau$ as a function of Re for the various fluids, as shown in Fig. 3. Note that the results are comparatively insensitive to the large discrepancies in Pr_t, once the Reynolds number is high enough for turbulent transport to dominate. Below $Re = 10^5$ in mercury and 4×10^4 in sodium, however, there is some uncertainty in $-\overline{v\theta}/u_\tau t_\tau$ and judgement has been used in selecting the most plausible variations, which are indicated by the dashed lines. For sodium, the variation deduced from the data of Fuchs [15] is followed, until it virtually coincides with that from Dwyer's correlation [25], which is then used for the higher Reynolds numbers. For mercury, Dwyer's correlation is abandoned in favour of the combined evidence of Hochreiter [12], Brown *et al.* [18], Sleicher *et al.* [19] and Isakoff and Drew [23]. For both fluids, if $Re > 4 \times 10^4$, estimated uncertainty in these best-fit lines is $\pm 10\%$.

It is remarked here in passing that the converse of the insensitivity of $-\overline{v\theta}/u_\tau t_\tau$ to Pr_t, is that small differences in the flow conditions between experiments, such as those caused by buoyancy-influence or probe interference, may be expected to result in large discrepancies between the measured values of Pr_t, as indeed has been observed.

Also plotted in Fig. 3 are the small variations with Re of $\overline{uv}/u_\tau^2$ and $-\overline{v\theta}/u_\tau t_\tau$ for $Pr > 10$. The former results from the decreased contribution of molecular viscosity to the total shear stress as Re increases. Since the molecular contribution to heat transport for $Pr > 10$ is negligible throughout the range of Re, the difference between the latter curve and 0.5 is due solely to the bowing of the heat-flux profile, as expressed by equation (2).

4. RESULTS FOR THE LEVEL OF RADIAL VELOCITY AND TEMPERATURE FLUCTUATION

The radial velocity fluctuations have been measured in isothermal air by many investigators and since, in the absence of effects due to imperfect wetting, the hydrodynamics of liquid metal systems should be identical, these values may be taken to apply in the absence of buoyancy influence. In the selected data reviewed by Lawn and White [28], no significant variation with Reynolds number was observed and $\tilde{v}/u_\tau = 0.95$ emerges as the average value at $r/a = 0.5$, with deviations of no more than $\pm 5\%$ in the isothermal experiments.

All the measurements known to the author of temperature fluctuations in pipe-flow are listed in the Table, with results quoted for $r/a = 0.5$. Those in air are discussed in [28] but those in liquid metals and aqueous compounds ($Pr > 1$) have not been collated previously. Most of the RMS values presented are not normalized by t_τ in the original papers, so this has been done using accepted friction factor and heat-transfer correlations in the formulae for u_τ and t_τ:

$$t_\tau \equiv \frac{\dot{q}_w''}{\rho u_\tau Cp} \equiv \frac{\dot{q}_w''}{\rho U_b Cp}\frac{U_b}{u_\tau} \equiv \frac{St}{(f/2)^{1/2}}(T_w - T_b) \quad (6)$$

The Blasius friction factor correlation was again adopted throughout, and the Lyon–Martinelli formula for Nusselt number (see [29]) used in interpreting the Russian data for mercury. There can be no certainty that the latter correlation did in fact apply but there is sufficient evidence to suggest that if this procedure did not yield t_τ to within $\pm 10\%$, then the experiment in question did not fulfil the requisite conditions of fully-developed flow with uniform heat-flux and negligible property variations. For air and water, the Dittus–Boelter correlation was used and again may be expected to have given t_τ correct to $\pm 10\%$.

5. DEDUCED RESULTS FOR THE CORRELATION COEFFICIENT

Dividing $-\overline{v\theta}/u_\tau t_\tau$ as given in Fig. 3 for the appropriate values of Re and Pr, by the measured values of $\tilde{v}/u_\tau$ and $\tilde{\theta}/t_\tau$ from the last section, results in the values of $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ quoted in Table 1.

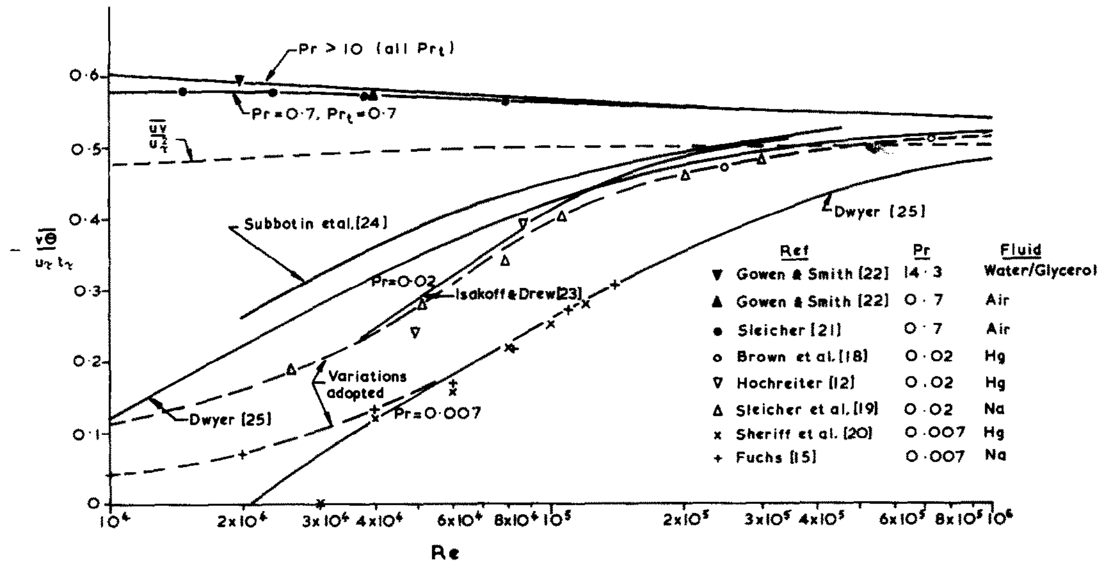

FIG. 3. Deduced variation of normalized turbulent heat flux with Reynolds number for various fluids at $r/a = 0.5$ in a pipe.

On the basis of the previously quoted uncertainties, the results for the correlation coefficient in the liquid metals with $Re > 4\times10^4$ should all be correct to $\pm 25\%$, if there were no error in $\tilde{\theta}$, or failure to set up the required conditions. In air and water, the results would be correct to $\pm 15\%$.

The only results for sodium are those of Fuchs [15]. It is remarkable that the deduced values of $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ are so consistent, yet at an impossible level greater than unity. The friction velocities used in estimating t_τ check with values derived by Fuchs himself and the values of Pr_t are also his. Only the physical properties of sodium at the prescribed temperature and $\tilde{v}/u_\tau$ were therefore assumed by this author in deriving $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ from $\tilde{\theta}/\dot{q}''_w$. Fuchs used guard-heaters to eliminate heat-losses from his test-section and since his measured Nusselt numbers lie between 20 and 30% below the Lyon–Martinelli formula, there is some, admittedly insubstantial, evidence to suggest that these heaters were imperfectly set and that his wall heat-flux was in fact higher than he calculated. Correction would reduce the $\tilde{\theta}/\dot{q}''_w$ values and give rise to correlation coefficients close to unity, which would be plausible, although still surprisingly high. Alternatively, flow disturbance of some kind may have increased $\tilde{v}$ above the fully-developed pipe flow value but in view of the great care evidently taken in these experiments, the results are paradoxical.

Impossible values, greater than unity, are also deduced from the data of Rust and Sesonske [30] for mercury. Hochreiter and Sesonske's work in NaK and mercury [12, 31, 32] indicated $-\overline{v\theta}/\tilde{v}\tilde{\theta} \sim 0.2$ but, because in mercury they in fact measured higher $-\overline{v\theta}/u_\tau t_\tau$ values than given by Fig. 3, their "measured" correlation coefficient (not quoted) must have been greater than 0.5. Results deduced from the work of Subbotin *et al.* [33] and Bobkov *et al.* [34] in mercury, although possible, are not sufficiently consistent to be useful.

The results for air based on the data of [1–3, 28, 35, 36] show greater consistency, although a wide range, $0.4 < -\overline{v\theta}/\tilde{v}\tilde{\theta} < 0.7$, is needed to include the majority of the results. No consistency at all appears in the high Prandtl number results [30, 37–39].

The only firm conclusion from this analysis of published results for temperature fluctuations, beyond the one that such measurements are extremely difficult, is therefore that for $Pr \sim 1$, $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ is somewhat larger than $\overline{uv}/\tilde{u}\tilde{v}$ at the one radial position examined, where the latter is between 0.36 and 0.42 [4]. The dependence of these correlation coefficients on radial position is similar and they vary little over 50% of the radius [1, 3, 4], so this conclusion is not restricted to $r/a = 0.5$.

6. CONCLUSION

Examination of experimental data for the spectral distribution of velocity and temperature fluctuations in the fully-turbulent region of heated pipe-flow has suggested a schematic representation which incorporates the essential features. The molecular "cut-off" points in air have been found to occur at a non-dimensional wave number $k_1 \simeq 0.1(\varepsilon/v^3)^{1/4}$ for velocity, and a somewhat higher value for temperature. Evidence has been cited to suggest that the $-v\theta$ correlation coefficient maintains higher values than the uv coefficient at wave-numbers in the inertial subrange. The theory of Batchelor *et al.* [8], and limited evidence from experiments in mercury, have then provided the form of the θ^2 and $-v\theta$ spectra in liquid metals, including rapid cut-off at $k_1 \sim (\varepsilon/\alpha^3)^{1/4}$. From this information, a limiting Peclet number (10^3) has been deduced, above which the correlation coefficient should be a weak function of Pe alone. Analysis shows that the variation in $-\overline{v\theta}/\tilde{v}\tilde{\theta}$ will be less than about $\pm 30\%$ for $Pe > 2\times10^3$.

An attempt to check this inference from published data for the R.M.S. level of temperature fluctuations

Table 1. Data for turbulent temperature fluctuations at $r/a = 0.5$

Ref.	Measuring instrument	Fluid	Pr	$Re \times 10^{-4}$	Method of deducing $\tilde{\theta}/t_\tau$	$\tilde{\theta}/t_\tau$	$-\overline{v\theta}/u_\tau t_\tau$ from Fig. 3	$-\overline{v\theta}/\tilde{v}\tilde{\theta}$ deduced	$-\overline{v\theta}/\tilde{v}\tilde{\theta}$ measured
Fuchs [15]	Thermocouple	Na	0.007	1.47	$\tilde{\theta}/\dot{q}''_w$	0.052	0.055	1.1	
				2.39		0.074	0.085	1.2	
				6.30		0.157	0.190	1.3	
				9.30		0.204	0.250	1.3	
				10.8		0.223	0.270	1.3	
				12.0		0.229	0.285	1.3	
				13.2		0.248	0.300	1.3	
				13.9		0.260	0.305	1.2	
Rust and Sesonske [30]	Thermocouple	Hg	0.02	14.5	$\tilde{\theta}$, $\dot{q}''_w$ given	0.33, 0.46	0.440	1.4, 1.0	
				24.3		0.29, 0.43	0.470	1.7, 1.1	
						(asymmetric)			
Hochreiter and Sesonske [31]	Thermocouple	NaK56	0.02	2.7	$\tilde{\theta}$, $\dot{q}''_w$ given	1.2, 1.1	0.190	0.17, 0.18	
				5.7		1.7, 1.2	0.300	0.19, 0.26	
						(asymmetric)			
Hochreiter and Sesonske [12, 32]	Cold-film	Hg	0.02	5.0	$\tilde{\theta}/t_\tau$ given in [12]	1.03	0.275 (0.45 measured)	0.28	0.53
Subbotin *et al.* [33]	Thermocouple	Hg	0.02	2.4	$Nu = 7+0.025Pe^{0.8}$,	1.24	0.180	0.15	
				21	and $\tilde{\theta}/(T_w - T_b)$ given	0.60	0.465	0.82	
Bobkov *et al.* [34]	Thermocouple	Hg	0.02	1.6	$Nu = 7+0.025Pe^{0.8}$, $\tilde{\theta}$ given $T_w - T_b$ deduced	0.36	0.145	0.42	
Tanimoto and Hanratty [35]	Thermocouple	Air	0.7	1.1	$\tilde{\theta}/t_\tau Pr$ given	0.90	0.580	0.68	
				3.9		1.05	0.575	0.57	
Lawn and White [28]	Cold-wire	Air	0.7	1.3	$\tilde{\theta}/t_\tau$ given	1.13	0.580	0.54	0.40
Carr *et al.* [36]	Hot-wire	Air	0.7	1.4	$\tilde{\theta}/t_\tau$ given	0.93	0.580	0.66	
Bourke and Pulling [2]	Cold-wire	Air	0.7	1.5	$\tilde{\theta}$, $\dot{q}''_w$ given	1.27	0.580	0.48	0.44
Ibragimov *et al.* [1]	Hot-wire	Air	0.7	3.2	$\tilde{\theta}/t_\tau Pr$ given	0.84	0.575	0.72	0.80
				26.0		0.82	0.555	0.71	0.66
Bremhorst and Bullock [3]	Cold-wire	Air	0.7	5.4	$\tilde{\theta}/t_\tau$ given	1.04	0.570	0.58	0.46
Bobkov *et al.* [37]	Thermocouple	H_2O	2.4–3.0	1.0	$St = St^*_{DB}$	0.27	0.605	2.3	
				2.0	and $\tilde{\theta}/(T_w - T_b)$ given	0.47	0.595	1.1	
				3.0		0.74	0.585	0.83	
Richard [38]	Thermocouple (ensemble variance)	H_2O	4.4–4.8	1.6	$\tilde{\theta}$, t_τ given	1.15	0.600	0.55	
				2.3		0.81	0.590	0.77	
				20		0.55–0.69	0.555	1.06–0.85	
Rust and Sesonske [30]	Thermocouple	Ethylene Glycol	44	1.3	$\tilde{\theta}$, $\dot{q}''_w$ given	0.33, 0.41 (asymmetric)	0.600	1.9, 1.5	
Kudva and Sesonske [39]	Cold-film	Ethylene Glycol	53	0.6	$\tilde{\theta}/t_\tau$ given	2.5	0.620	0.26	

$^*St_{DB} = 0.023Re^{-0.2}Pr^{-0.6}$.

in various fluids has proved inconclusive but several interesting results have emerged in the process.

(i) The turbulent Prandtl number at a given radial position is a function of Re and Pr by virtue of at least two different physical processes, and cannot therefore be expected to vary in a simple manner with either of these parameters or their product, the Peclet number.

(ii) Nevertheless, data for the turbulent Prandtl number in sodium do show a monotonic variation with Reynolds number. Data for mercury are more scattered but appear to vary in the same way as those for sodium.

(iii) The uncertainties in the turbulent Prandtl number data do not lead to large uncertainties ($< \pm 10\%$ for $Re > 4 \times 10^4$) in the estimated proportion of the radial heat-flux which is turbulent.

(iv) Although this proportion for sodium varies by a factor of 5 in the range $10^4 < Re < 10^5$, the results of Fuchs [15] for $\tilde{\theta}/\dot{q}''_w$ also show precisely this variation, so that the estimated correlation coefficient is indeed constant.

(v) In common with much of the data for fluids other than air, however, Fuch's results, taken in conjunction with data for $\tilde{v}$, imply that the correlation coefficients would have to be greater than unity to support the heat-flux. All these results for $\tilde{\theta}/\dot{q}''_w$ must therefore be in error.

(vi) The results for air show that $-\overline{v\theta}/\tilde{v}\tilde{\theta} > \overline{uv}/\tilde{u}\tilde{v}$ in the fully-turbulent region. This is due to the ability of the nearly isotropic subrange to transfer heat efficiently but not momentum.

(vii) The results for all fluids support the notion that $-\overline{v\theta}/\tilde{v}\tilde{\theta} = 0.5 \times\!\!\!\div 2$ over a large proportion of the radius and thus estimates of $\tilde{\theta}$ may be made to within a factor of 2, for uniform heating in a pipe.

Further measurements of temperature fluctuations in all fluids with particular attention to measuring the true wall heat-flux, are however required and should be subjected to the simple check provided by the present analysis. Such data would be invaluable in the development of the theory of turbulent heat transport.

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APPENDIX

Anticipated Behaviour of the Heat-Flux Correlation Coefficient in Low Prandtl Number Fluids

Let $R_\infty \equiv \overline{v\theta}_\infty/\sqrt{(\overline{v^2}\,\overline{\theta^2_\infty})}$ be the value of the correlation coefficient at a given radial position and at sufficiently high Peclet number for it to be invariant (according to the theory of the main text) and let $R_{v\theta}$ be its value in a low Prandtl number fluid, for which nevertheless $a\eta_c^{-1} > 6\pi$ at the Reynolds number considered. Then

$$R_{v\theta} \simeq \frac{\int_0^{\eta_c^{-1}} E_{v\theta}(k_1)\,dk_1}{\left\{\int_0^{\infty} E_{v^2}(k_1)\,dk_1 \int_0^{\eta_c^{-1}} E_{\theta^2}(k_1)\,dk_1\right\}^{1/2}} \quad \text{(A1)}$$

since both $E_{v\theta}$ and E_{θ^2} decay rapidly for $k_1 > \eta_c^{-1}$. Let β and γ be defined by:

$$\overline{\theta^2} = \int_0^{\eta_c^{-1}} E_{\theta^2}\,dk_1 \equiv (1-\beta)\overline{\theta^2_\infty} \quad \text{(A2)}$$

and

$$\int_0^{\eta_c^{-1}} E_{v^2}dk_1 = (1-\gamma)\overline{v^2}. \quad \text{(A3)}$$

Now

$$\int_{\eta_c^{-1}}^{\infty} E_{v\theta\infty}(k_1)\,dk_1 < \beta^{1/2}\gamma^{1/2}|\overline{v\theta}_\infty|,$$

since the correlation coefficient falls off at high wave-number, so

$$|\overline{v\theta}_\infty| > \left|\int_0^{\eta_c^{-1}} E_{v\theta}(k_1)\,dk_1\right| > |\overline{v\theta}_\infty|(1-\beta^{1/2}\gamma^{1/2}). \quad \text{(A4)}$$

Thus

$$\frac{1}{(1-\beta)^{1/2}} > \left|\frac{R_{v\theta}}{R_\infty}\right| > \frac{(1-\beta^{1/2}\gamma^{1/2})}{(1-\beta)^{1/2}} \quad \text{(A5)}$$

and since $\gamma > \beta$, see Fig. 1(a) and [3, 4],

$$\frac{1}{(1-\gamma)^{1/2}} > \left|\frac{R_{v\theta}}{R_\infty}\right| > 1-\gamma. \quad \text{(A6)}$$

Upon integration of the $E_{v^2}(k_1)$ spectrum in Fig. 1(a), it is seen that:

$$\gamma = \frac{3}{5}\left\{\frac{(\varepsilon/\alpha^3)^{1/4}}{6\pi}\right\}^{-2/3}, \quad \text{(A7)}$$

so for $Pe = 2\times10^3$ ($Re = 4\times10^5$ in sodium), giving $a\eta_c^{-1} \simeq 1.6(a^+Pr)^{3/4} = 12\pi$, we have

$$1.27 > \left|\frac{R_{v\theta}}{R_\infty}\right| > 0.62, \quad \text{(A8)}$$

these limits almost certainly being wider than in reality. Thus no more than ~30% changes in the correlation coefficient from the high Peclet number situation are to be expected.

FLUCTUATIONS TURBULENTES DE TEMPERATURE DANS LES METAUX LIQUIDES

Résumé—L'examen des résultats expérimentaux sur la distribution spectrale des fluctuations de vitesse (u et v) et de température (θ), dans la région pleinement turbulente d'un écoulement dans un tube chauffé, a suggéré une représentation schématique qui inclut les caractères essentiels. L'observation est utilisée pour dire que le coefficient de corrélation "$-v\theta$" a des valeurs plus élevées que le coefficient "uv", aux nombres d'onde dans le domaine inertiel. La théorie de Batchelor, Howells et Townsend et l'observation d'expériences dans le mercure suggèrent la forme des spectres de "θ^2" et "$-v\theta$" dans les métaux liquides. A partir de cette information, on déduit un nombre de Péclet limite, au dessus duquel le coefficient de corrélation de v et de θ peut être très faiblement fonction de Pe seul.

Un essai de cette inférence à partir des données publiées sur les valeurs quadratiques moyennes des fluctuations de température et sur les nombres de Prandtl turbulents n'est pas concluant parce que beaucoup de coefficients de corrélation ainsi estimé ont des valeurs supérieures à l'unité. On conclut que tous ces résultats sur $\tilde{\theta}$ sont entachés d'erreur. Néanmoins, puisqu'il n'y a pas de coefficients de corrélations très faibles, ils se situent certainement principalement dans le domaine 0,5 × 2 pour une grande proportion du rayon. Par suite $\tilde{\theta}$ peut être estimé pour un fluide quelconque, dans lequel les fluctuations sont induites par un chauffage uniforme, au moins avec un facteur 2 à partir de cette analyse.

TURBULENTE TEMPERATURSCHWANKUNGEN IN FLÜSSIGEN METALLEN

Zusammenfassung—Die Auswertung experimenteller Daten für die Spektralverteilung der Geschwindigkeitsschwankungen (u und v) und der Temperaturschwankungen (θ) bei voll ausgebildeter turbulenter Rohrströmung mit Wärmezufuhr legt eine schematische Darstellung nahe, welche die wesentlichen Merkmale beinhaltet. Aufgrund der angeführten Beweise kann vermutet werden, daß bei Wellenzahlen im Trägheitsbereich der "$-v\theta$"-Korrelationskoeffizient höhere Werte besitzt als der "uv"-Koeffizient. Die Theorie von Batchelor, Howells und Townsend sowie mit Einschränkungen Versuchsergebnisse mit Quecksilber lassen sodann auf die Form der "θ^2"- und "$-v\theta$"-Spektren in flüssigen Metallen schließen. Hieraus wird eine Grenz-Péclet-Zahl abgeleitet, oberhalb welcher der Korrelationskoeffizient für v und θ nur noch eine schwache Funktion der Péclet-Zahl allein sein sollte.

Der Versuch, diesen Tatbestand anhand veröffentlichter Daten für die quadratisch gemittelten Werte der Temperaturschwankungen und für die turbulenten Prandtl-Zahlen zu überprüfen, erweist sich als unschlüssig, da viele der so geschätzten Korrelationskoeffizienten Werte größer als eins besitzen. Hieraus wird gefolgert, daß alle diese Ergebnisse für $\tilde{\theta}$ fehlerhaft sein müssen. Da jedoch sehr kleine Korrelationskoeffizienten nicht auftreten, liegen sie für einen weiten Rechenbereich sicherlich im Bereich zwischen 0,5 und 2. Damit läßt sich für alle Fluide, bei denen die Schwankungen durch gleichmäßige Wärmezufuhr bedingt sind, der Wert $\tilde{\theta}$ wenigstens bis auf einen Faktor 2 genau mit Hilfe der vorgeschlagenen Methode bestimmen.

ТУРБУЛЕНТНЫЕ ПУЛЬСАЦИИ ТЕМПЕРАТУРЫ В ЖИДКИХ МЕТАЛЛАХ

Аннотация — В результате анализа экспериментальных данных по спектральному распределению пульсаций скорости (u и v) и температуры (θ) в полностью развитой турбулентной области течения в нагретой трубе предложена схема формирования основных характеристик процесса. Приведены данные, свидетельствующие о том, что при волновых числах в инерционном интервале значения коэффициента корреляции «$-v\theta$» превышают значения коэффициента «uv». В соответствии с теорией Бэтчелора, Хоуэллса и Таусенда и экспериментальными данными, полученными на ртути, предложена форма спектра θ^2 и взаимного спектра «$-v\theta$» в жидких металлах. На основании этого получено предельное значение числа Пекле, при превышении которого коэффициент корреляции v и θ является довольно слабой функцией только числа Пекле. Данный результат не получил подтверждения при его сопоставлении с известными экспериментальными данными для среднего квадрата пульсаций тепмературы и для турбулентного чилса Прандтля, так как значения большинства из оцененных таким образом коэффициентов корреляций превышали единицу. Отсюда очевидна ошибочность результатов, полученных для θ. Однако, в связи с тем, что низких значений коэффициентов корреляции получено не было, они для большей части радиуса лежат скорее всего в диапазоне 0.5–2. Таким образом, пользуясь результатами настоящей работы, можно, по крайней мере с точностью до 50%, определить значение θ для любой жидкости, в которой пульсации температуры создаются при равномерном нагревании.